

END OF SEMESTER EXAMINATIONS, APRIL/MAY - 2017

CALCULUS

SUBJECT CODE: 08UAMA01

MAJOR: B.Sc. MATHS

TIME : 3 HOURS

SEMESTER : I

MAX. MARKS: 75

SECTION -A
MARKS: 5X2=10

Answer all questions

1. Write the equation of sub tangent in Cartesian coordinates.
or
2. Write the equation of radius of curvature when the curve is given in polar coordinates.
3. Find the asymptotes of $(x+y)(x-y)(x-2y-4) = 3x+7y-6$.
or
4. Write the practical rule to find double points.
5. Say whether $\int_0^{\infty} e^x dx$ converges, diverges or oscillates?
or
6. $\int_0^{\pi/2} \sin^6 x \cos^5 x dx = ?$
7. Find the area enclosed by the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$.
or
8. Write the transformation from Cartesian to spherical polar coordinates.
9. If $f(x)$ is even, then $\int_{-a}^a f(x) dx = ?$.
or
10. Define an even Function.

SECTION -B
MARKS: 5X4=20

Answer all questions

11. Show that in the curve $r = e^{\theta \cot \alpha}$ (i) the polar sub tangent $= r \tan \alpha$ (ii) the polar sub normal $= r \cot \alpha$.
or
12. Derive the Cartesian formula for the radius of curvature.
13. Find the asymptotes of the $x^3 + 2x^2y - xy^2 - 2y^3 + 4y^2 + 2xy + y - 1 = 0$.
or
14. Show that the curve $y^2 = 2x^2y + x^4y + x^4$ has a double keratoid cusp at the origin.
15. Evaluate $\int_0^{\infty} e^{-x^2} dx$.
or
16. If $f(x)$ is continuous in (a, b) , then prove that at every point x in (a, b) , $F(x)$ has a derivative equal to $f(x)$.
17. By changing the order of integration, evaluate $\int_0^{\infty} \int_x^{\infty} \frac{e^{-y}}{y} dx dy$.
or
18. Find the area of the curvilinear quadrilateral bounded by the four parabolas $y^2 = ax, y^2 = bx, x^2 = cy, x^2 = dy$.
19. Express $f(x) = c - x$ where $0 < x < c$ as a half range cosine series with period $2c$.
or
20. Express $f(x) = x$ ($-\pi < x < \pi$) as a Fourier series with period 2π .

SECTION -C
MARKS: 5X9=45

Answer all questions

21. Derive the coordinates of centre of curvature of the curve $y = f(x)$.

or

22. Show that the evolute of the cycloid $x = a(\theta - \sin\theta)$; $y = a(1 - \cos\theta)$ is another cycloid.

23. Find the asymptotes of $x^3 + 2x^2y - 4xy^2 - 8y^3 - 4x + 8y = 1$,

or

24. Trace the curve $(a^2 + x^2)y = a^2x$.

25. Express $\int_0^1 x^m (1 - x^n)^p dx$ in terms of Gamma functions and evaluate the integral

$$\int_0^1 x^5 (1 - x^3)^{10} dx.$$

or

26. Given $I_{m,n} = \int x^m (\log x)^n dx$ (m, n being positive integers), evaluate $\int x^4 (\log x)^3 dx$.

27. Evaluate $\iiint xyz dx dy dz$ taken through the positive octant of the sphere $x^2 + y^2 + z^2 = a^2$.

or

28. Given that $x + y = u$, $y = uv$, change the variables to u, v in the integral $\iint [xy(1 - x - y)]^{\frac{1}{2}} dx dy$ taken over the area of the triangle with sides $x = 0, y = 0, x + y = 1$ and evaluate it.

29. Express $f(x) = \frac{1}{2} (\pi - x)$ as a Fourier series with period 2π , to be valid in the interval 0 to 2π .

or

30. Find a sine and a cosine series for the function $f(x) = 3x - 2$ in the interval $0 < x < 4$.