

END OF SEMESTER EXAMINATIONS, NOVEMBER - 2017

CALCULUS

SUBJECT CODE: 08UAMA01

MAJOR: B.Sc.,(Mathematics)

TIME : 3 HOURS

SEMESTER: I

MAX.MARKS: 75

SECTION - A (5 X 2 = 10)**Answer ALL the Questions:**

1. At which point on the curve
- $y = x^3 - 12x + 18$
- is the tangent parallel to the
- x
- axis?

(OR)

2. Find
- $\frac{ds}{dr}$
- for the Cardioid
- $r = a(1 + \cos \theta)$

3. Write the rule to find asymptotes parallel to the axes.

(OR)

4. Write the practical rule to determine double points

5. Evaluate
- $\int_0^{\pi/2} \cos^8 x \, dx$
- .

(OR)

6. Define the Beta and Gamma functions.

7. Evaluate
- $\iint xy \, dx \, dy$
- taken over the positive quadrant of the circle
- $x^2 + y^2 = a^2$
- .

(OR)

8. Write the Jacobian determinant of any two variables.

9. Write the Euler's formulae in Fourier Series.

(OR)

10. Define even and odd functions with examples.

SECTION - B (5 X 4 = 20)**Answer ALL the Questions:**

11. What is the direction of the tangent at (2,1) to the curve
- $x^3 + y^3 = 9$
- ?

(OR)

12. Find the angle at which the radius vector cuts the curve
- $\frac{r}{a} = 1 + e \cos \theta$
- .

13. Find the asymptotes of
- $x^3 + 2x^2y - xy^2 - 2y^3 + 4y^2 + 2xy + y - 1 = 0$
- .

(OR)

14. Show that for conicoid
- $x^2y^2 = (a+y)^2(b^2 - y^2)$
- if
- $b > a$
- , there is a node at
- $x=0, y=-a$
- and if
- $y=-a$
- and if
- $b=a$
- there is a cusp at the same point.

15. Evaluate
- $\int_0^{\pi} \theta \sin^5 \theta \, d\theta$
- .

(OR)

16. Express
- $\int_0^1 x^m (1-x^n)^p \, dx$
- in terms of Gamma functions and evaluate the integral

$$\int_0^1 x^5 (1-x^3)^{10} \, dx.$$

17. Evaluate the double integral $\iint_R (4 - x^2 - y^2) dx dy$ if the region R is bounded by the straight lines $x = 0$, $x = 1$, $y = 0$ and $y = \frac{3}{2}$.
(OR)

18. By changing the order of integration, evaluate $\int_0^\infty \int_x^\infty \frac{e^{-y}}{y} dx dy$.

19. Show that the Fourier Series for $f(x) = x$, $-\pi < x < \pi$

is given by $f(x) = 2 \sum_{n=1}^{\infty} (-1)^{n+1} \cdot \frac{\sin nx}{n}$.
(OR)

20. Obtain the sine series for the function $f(x) = \begin{cases} x & \text{in } 0 \leq x \leq \frac{1}{2} \\ 1-x & \text{in } \frac{1}{2} \leq x \leq 1 \end{cases}$.

SECTION - C (5 X 9 = 45)

Answer ALL the Questions:

21. Find the envelope of the straight lines $\frac{x}{a} + \frac{y}{b} = 1$ where the parameters are related by the equation $a^2 + b^2 = c^2$, where C is a constant.
(OR)

22. Find the Evolute of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$.

23. Determine the asymptotes of the curve $4(x^4 + y^4) - 17x^2y^2 - 4x(4y^2 - x^2) + 2(x^2 - 2) = 0$ and show that they pass through the points of intersection of the curve with the ellipse $x^2 + 4y^2 = 4$.
(OR)

24. Trace the curve $y = (x-1)(x-2)(x-3)$.

25. Evaluate $\int_0^\pi \frac{x \tan x}{\sec x + \tan x} dx$.
(OR)

26. Derive the relation between Beta and Gamma functions.

27. Change the order of integration and hence evaluate $\int_0^{4a} \int_{x^2/4a}^{2\sqrt{ax}} xy dy dx$.
(OR)

28. Evaluate $\int_0^{2\pi} \int_0^{\pi/4} \int_0^a r^2 \sin \theta dr d\theta d\phi$ since.

29. Find the Fourier Series for the function $f(x) = e^x$ defined in $(-\pi, \pi)$.
(OR)

30. Find a Cosine Series for the function $f(x) = \begin{cases} x & \text{in } 0 \leq x < \pi/2 \\ \pi - x & \text{in } \pi/2 \leq x < \pi \end{cases}$
