

END OF SEMESTER EXAMINATIONS, APRIL / MAY -2018
 TRIGONOMETRY, VECTOR CALCULUS AND FOURIER TRANSFORMS
 SUBJECT CODE: 08UAMA04

MAJOR: B.Sc.,(Mathematics)
 TIME : 3 HOURS

SEMESTER: II
 MAX.MARKS: 75

SECTION - A (5 X 2 = 10)

Answer ALL Questions:

1. Expand $\tan n\theta$ in powers of $\tan \theta$.
 (OR)
2. Prove that $\cosh 2x = \cosh^2 x + \sinh^2 x$
3. Find $\log(1-i)$.
 (OR)
4. Define Logarithms of complex quantities.
5. If $\phi(x, y, z) = x^2y - 2y^2z^3$ Find $\nabla\phi$ at $(1, -1, 2)$.
 (OR)
6. Show that $\vec{F} = 3y^4z^2\vec{i} + 4x^3z^2\vec{j} - 3x^2y^2\vec{k}$ is Solenoidal.
7. If $\vec{F} = yz\vec{i} + zx\vec{j} - xy\vec{k}$ Find $\int_c \vec{F} \cdot d\vec{r}$ where c is given by $x = t, y = t^2, z = t^3$ from $(0, 0, 0)$ to $(2, 4, 8)$.
 (OR)
8. State Gauss divergence theorem.
9. Define Fourier sine transform of $f(t)$.
 (OR)
10. Define convolution of two functions.

SECTION - B (5 X 4 = 20)

Answer ALL Questions:

11. Express $\cos 8\theta$ in terms of $\sin \theta$.
 (OR)
12. If $\sin(A+iB) = x+iy$ prove that (i) $\frac{x^2}{\sin^2 A} - \frac{y^2}{\cos^2 A} = 1$ (ii) $\frac{x^2}{\cosh^2 B} + \frac{y^2}{\sinh^2 B} = 1$,
13. Show that $\log_i = \frac{4n+1}{4m+1}$ where m and n are integers.
 (OR)
14. Prove that $\log \frac{a+ib}{a-ib} = 2i \tan^{-1}\left(\frac{b}{a}\right)$.
15. Find the value of 'a' such that $\vec{F} = (axy - z^2)\vec{i} + (x^2 + 2yz)\vec{j} + (y^2 - axz)\vec{k}$ is irrotational.
 (OR)
16. Prove that $\nabla \cdot [\nabla r^n] = n(n+1)r^{n-2}$.
17. Find the total work done in moving a particle in a force field given by

$$\vec{F} = 3xy\vec{i} - 5z\vec{j} + 10z\vec{k}$$
 along the curve $x = t^2 + 1, y = 2t^2, z = t^3$ from $t = 1$ to $t = 2$.
 (OR)
18. Show that $\iint_s \vec{r} \cdot \vec{n} ds = 3V$ where V is the volume enclosed by the closed surface s .

19. Find the Fourier Transform of

$$f(t) = \begin{cases} t; & |t| < a \\ 0; & |t| > a \end{cases}$$

(OR)

20. Find Fourier sine transform of te^{-at} .

SECTION - C (5 X 9 = 45)

Answer ALL Questions:

21. Expand $\sin^3 \theta \cos^4 \theta$ in terms of Sines of multiples of θ .

(OR)

22. If $\cos(x+iy) = r(\cos \alpha + i \sin \alpha)$ prove that $y = \frac{1}{2} \log \left[\frac{\sin(x-\alpha)}{\sin(x+\alpha)} \right]$

23. Show that $\log \tan \left(\frac{\pi}{4} + i \frac{x}{2} \right) = i \tan^{-1} (\sinh x)$.

(OR)

24. If $\log \sin(\theta + i\varphi) = A + iB$ show that

(i) $2e^{2A} = \cosh 2\phi - \cos 2\theta$

(ii) $\cos(\theta - B) = e^{2\phi} \cos(\theta + B)$

25. Prove that $\nabla \cdot \left[\frac{f(r)}{r} \vec{r} \right] = \frac{1}{r^2} \frac{d}{dr} (r^2 f)$.

(OR)

26. A Field \vec{F} is of the form $\vec{F} = (6xy + z^3)\vec{i} + (3x^2 - z)\vec{j} + (3xz^2 - y)\vec{k}$. Show that \vec{F} is conservative field and find a function ϕ such that $\vec{F} = \nabla \phi$.

27. Evaluate $\iint_S \vec{F} \cdot \vec{n} ds$ where $\vec{F} = z\vec{i} + x\vec{j} - 3y^2z\vec{k}$ and s is surface of the cylinder $x^2 + y^2 = 16$ included in the first octant between $z=0$ and $z=5$.

(OR)

28. Using Divergence theorem evaluate $\int_S \vec{F} \cdot \vec{n} ds$ where $\vec{F} = 4xz\vec{i} - y^2\vec{j} + yz\vec{k}$ and s the surface of the cube bounded by the planes $x=0, x=2, y=0, y=2, z=0, z=2$

29. Find the Fourier transform of $f(x) = \begin{cases} 1-x^2; & |x| \leq 1 \\ 0; & |x| \geq 1 \end{cases}$ Hence show

that $\int_0^\infty \left(\frac{\sin s - s \cos s}{s^3} \right) \cos \frac{s}{2} ds = \frac{3\pi}{16}$.

(OR)

30. Find Fourier cosine transform of e^{-at} hence evaluate $\int_0^\infty \frac{\cos at}{x^2 + a^2} dx$.
