

**END OF SEMESTER EXAMINATIONS, NOVEMBER - 2018**  
**DIFFERENTIAL EQUATIONS & LAPLACE TRANSFORMS**  
**SUBJECT CODE: 16UAMA03**

**MAJOR: B.Sc.,(Mathematics)**  
**TIME : 3 HOURS**

**SEMESTER : II**  
**MAX MARKS : 75**

**SECTION - A ( 5 X 2 = 10 )**

**Answer ALL Questions:**

1. Solve:  $x = y^2 = \log p$ .  
 (OR)

2. Solve:  $\frac{d^3y}{dx^3} - 3 \frac{dy}{dx} + 2y = 0$ .

3. Find the particular integral of  $x^2 \frac{d^2y}{dx^2} + 4x \frac{dy}{dx} + 2y = e^x$ .  
 (OR)

4. Find the complementary function of  $x^2 \frac{d^2y}{dx^2} + 3x \frac{dy}{dx} + y = \frac{1}{(1-x)^2}$ .

5. Solve:  $\frac{\partial^2 z}{\partial y^2} = \sin y$ .  
 (OR)

6. Solve:  $p = y^2 q^2$ .

7. Find  $L(te^{-at})$ .  
 (OR)

8. Find  $L^{-1}\left[\frac{s}{s^2 + k^2}\right]$ .

9. What is the general solution of Lagrange's equation?  
 (OR)

10. Write the auxiliary equation of  $(p-q)x + y(px+qy) = yz$ .

**SECTION - B ( 5 X 4 = 20 )**

**Answer ALL Questions:**

11. Solve:  $y^2 = 2pxy + y^2 p^2$ .  
 (OR)

12. Solve:  $(D^2 + 5D + 6)y = e^x$ .

13. Solve:  $\frac{dx}{dy} = \frac{dy}{xz} = \frac{dz}{xy}$ .  
 (OR)

14. Solve:  $3x^2 \frac{d^2y}{dx^2} + x \frac{dy}{dx} + y = x$ .

15. Find the partial differential equation of all spheres whose centres lie on the plane  $z=0$  and whose radius is constant and equal to  $r$ .  
 (OR)

16. Solve:  $\frac{\partial^2 z}{\partial x^2} - 4 \frac{\partial z}{\partial x} + 3z = e^{2x}$ .

17. Find  $L\left(\frac{\sin at}{t}\right)$ .  
 (OR)

18. Find  $L^{-1}\left[\frac{s}{(s^2 + a^2)^2}\right]$ .

19. Solve:  $xp - yq = z$ .

(OR)

20. Find the general solution of  $(y+z)p + (z+x)q = x+y$ .SECTION - C (5 X 9 = 45)Answer ALL Questions:21. Show that the solution of the differential equation  $\frac{d^2y}{dt^2} + 4y = A \sin pt$  which is such that

$$y=0 \text{ & } \frac{dy}{dt}=0 \text{ when } t=0$$

$$y = \begin{cases} A \left( \frac{\sin pt - \frac{1}{2} p \sin 2t}{4-p^2} \right) & \text{if } p \neq 2 \\ A \left( \frac{\sin 2t - 2t \cos 2t}{8} \right) & \text{if } p = 2 \end{cases}$$

(OR)

22. a) Solve  $y = xp + x(1+p^2)^{1/2}$  [4 Marks]b) Solve:  $(xp+y)^2 = a(1+p^2)\phi(x^2+y^2)$  [5 Marks]23. Solve:  $\frac{d^2y}{dx^2} + y = \sec x$ .

(OR)

24. a) Solve:  $\frac{dx}{xy} = \frac{dy}{y^2} = \frac{dx}{x(yz-2x)}$  [6 Marks]b) Solve:  $x^2 \frac{d^2y}{dx^2} + x \frac{dy}{dx} + y = \log x$  [3 Marks]25. By the substitution  $u = x + \alpha y$ ,  $v = x + \beta y$ , where  $\alpha$  and  $\beta$  are two suitable constants,transform the partial differential equation  $\frac{\partial^2 z}{\partial x^2} - 5 \frac{\partial^2 z}{\partial x \partial y} + 6 \frac{\partial^2 z}{\partial y^2} = 0$  to the form  $\frac{\partial^2 z}{\partial u \partial v} = 0$  and

hence obtain its complete solution.

(OR)

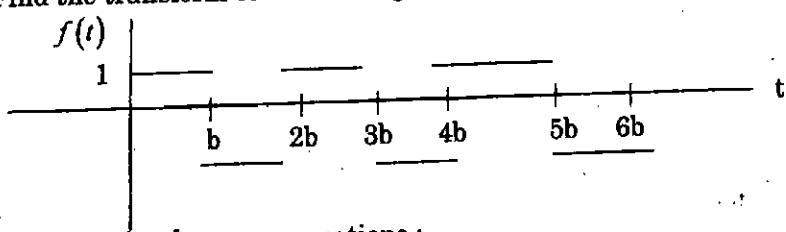
26. a) Eliminate the arbitrary functions  $f$  and  $\phi$  from the relation  $z = f(x+ay) + \phi(x-ay)$ 

[5 Marks]

b) Solve:  $\frac{\partial^2 z}{\partial x^2} - 4 \frac{\partial z}{\partial x} + 3z = e^{2x}$  [4 Marks]27. a) Find  $L^{-1} \left[ \frac{1+2s}{(s+2)^2(s-1)^2} \right]$  [6 Marks] b) Evaluate  $\int e^{-2t} \sin 3t dt$  [3 Marks]

(OR)

28. Find the transform of the rectangular wave shown below.



29. Solve the simultaneous equations :

$$\frac{dx}{dt} - \frac{dy}{dt} - 2x + 2y = 1 - 2t$$

$$\frac{d^2x}{dt^2} + 2 \frac{dy}{dt} + x = 0$$

with the conditions  $x = 0$ ,  $y = 0$ ,  $\frac{dx}{dt} = 0$  when  $t = 0$ .

(OR)

30. Solve  $\frac{d^2y}{dt^2} + 2 \frac{dy}{dt} - 3y = \sin t$  given that  $y = \frac{dy}{dt} = 0$  when  $t = 0$ .

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