

END OF SEMESTER EXAMINATIONS, APRIL / MAY -2018  
 TRIGONOMETRY, VECTOR CALCULUS AND FOURIER TRANSFORMS  
 SUBJECT CODE: 08UAMA04

MAJOR: B.Sc.,(Mathematics)  
 TIME : 3 HOURS

SEMESTER: II  
 MAX.MARKS: 75

SECTION – A (5 X 2 = 10)

**Answer ALL Questions:**

1. Expand  $\tan n\theta$  in powers of  $\tan \theta$ .  
 (OR)
2. Prove that  $\cosh 2x = \cosh^2 x + \sinh^2 x$
3. Find  $\log(1-i)$ .  
 (OR)
4. Define Logarithms of complex quantities.
5. If  $\phi(x, y, z) = x^2y - 2y^2z^3$  Find  $\nabla\phi$  at  $(1, -1, 2)$ .  
 (OR)
6. Show that  $\vec{F} = 3y^4z^2\vec{i} + 4x^3z^2\vec{j} - 3x^2y^2\vec{k}$  is Solenoidal.
7. If  $\vec{F} = yz\vec{i} + zx\vec{j} - xy\vec{k}$  Find  $\int_C \vec{F} \cdot d\vec{r}$  where  $C$  is given by  $x = t, y = t^2, z = t^3$  from  $(0, 0, 0)$  to  $(2, 4, 8)$ .  
 (OR)
8. State Gauss divergence theorem.
9. Define Fourier sine transform of  $f(t)$ .  
 (OR)
10. Define convolution of two functions.

SECTION – B (5 X 4 = 20)

**Answer ALL Questions:**

11. Express  $\cos 8\theta$  in terms of  $\sin \theta$ .  
 (OR)
12. If  $\sin(A+iB) = x+iy$  prove that (i)  $\frac{x^2}{\sin^2 A} - \frac{y^2}{\cos^2 A} = 1$  (ii)  $\frac{x^2}{\cosh^2 B} + \frac{y^2}{\sinh^2 B} = 1$ .
13. Show that  $\log_i = \frac{4n+1}{4m+1}$  where  $m$  and  $n$  are integers.  
 (OR)
14. Prove that  $\log \frac{a+ib}{a-ib} = 2i \tan^{-1}\left(\frac{b}{a}\right)$ .
15. Find the value of 'a' such that  $\vec{F} = (axy - z^2)\vec{i} + (x^2 + 2yz)\vec{j} + (y^2 - axz)\vec{k}$  is irrotational.  
 (OR)
16. Prove that  $\nabla \cdot [\nabla r^n] = n(n+1)r^{n-2}$ .
17. Find the total work done in moving a particle in a force field given by  

$$\vec{F} = 3xy\vec{i} - 5z\vec{j} + 10z\vec{k}$$
 along the curve  $x = t^2 + 1, y = 2t^2, z = t^3$  from  $t = 1$  to  $t = 2$ .  
 (OR)
18. Show that  $\iint_S \vec{r} \cdot \vec{n} ds = 3V$  where  $V$  is the volume enclosed by the closed surface  $S$ .

19. Find the Fourier Transform of

$$f(t) = \begin{cases} t; & |t| < a \\ 0; & |t| > a \end{cases}$$

(OR)

20. Find Fourier sine transform of  $te^{-at}$ .

**SECTION - C (5 X 9 = 45)**

**Answer ALL Questions:**

21. Expand  $\sin^3 \theta \cos^4 \theta$  in terms of Sines of multiples of  $\theta$ .

(OR)

22. If  $\cos(x+iy) = r(\cos \alpha + i \sin \alpha)$  prove that  $y = \frac{1}{2} \log \left[ \frac{\sin(x-\alpha)}{\sin(x+\alpha)} \right]$

23. Show that  $\log \tan \left( \frac{\pi}{4} + i \frac{x}{2} \right) = i \tan^{-1} (\sinh x)$ .

(OR)

24. If  $\log \sin(\theta + i\varphi) = A + iB$  show that

(i)  $2e^{2A} = \cosh 2\phi - \cos 2\theta$

(ii)  $\cos(\theta - B) = e^{2\phi} \cos(\theta + B)$

25. Prove that  $\nabla \cdot \left[ \frac{f(r)}{r} \hat{r} \right] = \frac{1}{r^2} \frac{d}{dr} (r^2 f)$ .

(OR)

26. A Field  $\vec{F}$  is of the form  $\vec{F} = (6xy + z^3)\vec{i} + (3x^2 - z)\vec{j} + (3xz^2 - y)\vec{k}$ . Show that  $\vec{F}$  is conservative field and find a function  $\phi$  such that  $\vec{F} = \nabla \phi$ .

27. Evaluate  $\iint_S \vec{F} \cdot \vec{n} ds$  where  $\vec{F} = z\vec{i} + x\vec{j} - 3y^2\vec{k}$  and  $S$  is surface of the cylinder

$$x^2 + y^2 = 16 \text{ included in the first octant between } z=0 \text{ and } z=5.$$

(OR)

28. Using Divergence theorem evaluate  $\int_S \vec{F} \cdot \vec{n} ds$  where  $\vec{F} = 4xz\vec{i} - y^2\vec{j} + yz\vec{k}$  and  $S$  the surface of the cube bounded by the planes  $x=0, x=2, y=0, y=2, z=0, z=2$

29. Find the Fourier transform of  $f(x) = \begin{cases} 1-x^2; & |x| \leq 1 \\ 0; & |x| \geq 1 \end{cases}$  Hence show

$$\text{that } \int_0^\infty \left( \frac{\sin s - s \cos s}{s^3} \right) \cos \frac{s}{2} ds = \frac{3\pi}{16}.$$

(OR)

30. Find Fourier cosine transform of  $e^{-at}$  hence evaluate  $\int_a^\infty \frac{\cos at}{x^2 + a^2} dx$ .

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