

S.No. 430

BATCH: 87 - 2013, 2016

END OF SEMESTER EXAMINATIONS, APRIL / MAY 2017  
 TRIGONOMETRY, VECTOR CALCULUS AND FOURIER TRANSFORMS  
 SUBJECT CODE: 16UAMA04

MAJOR: B.Sc. MATHS  
 TIME : 3 HOURS

SEMESTER : II  
 MAX.MARKS: 75

**SECTION - A ( 5 X 2 = 10 )**

**Answer ALL Questions:**

1. Write down the expansion of  $\sin 4\theta$  in terms of  $\sin \theta$  and  $\cos \theta$ .  
 (OR)
2. Find the real and imaginary part of  $\sinh(\alpha + i\beta)$ .
3. Find  $\log(2i)$ .  
 (OR)
4. Prove  $\log[(1+xi)/(1-xi)] = 2i \tan^{-1}x$ .
5. Find a unit vector normal to the surface  $x^2 + y^2 - z^2 = 1$  at  $(1, 1, 1)$ .  
 (OR)
6. If  $\vec{r} = x\vec{i} + y\vec{j} + z\vec{k}$  then find  $\operatorname{div} \vec{r}$ .
7. Evaluate  $\int \vec{F} \cdot d\vec{r}$  along the line  $y = x$  from  $(0, 0)$  to  $(1, 1)$  where  $\vec{F} = x^2\vec{i} + y^2\vec{j}$ .  
 (OR)
8. State Greens' theorem.
9. Find the Fourier Transform of  $f(t)$  where  $f(t) = \begin{cases} a & , -\ell < t < 0 \\ 0 & \text{otherwise} \end{cases}$   
 (OR)
10. State convolution theorem.

**SECTION - B ( 5 X 4 = 20 )**

**Answer ALL Questions:**

11. Find the real part of  $\tan^{-1}(\alpha + i\beta)$ .  
 (OR)
12. Expand  $\sin^3 \theta \cdot \cos^5 \theta$  in a series of multiples of  $\theta$ .
13. If  $\tan[\log(x+iy)] = a+ib$  then show that  $\tan[\log(x^2+y^2)] = \frac{2a}{1-(a^2+b^2)}$ .  
 (OR)
14. Sum to infinity the series  $c \sin \alpha + \frac{c^2}{2!} \sin 2\alpha + \frac{c^3}{3!} \sin 3\alpha + \dots$
15. If  $\vec{f} = x^2y\vec{i} + xz\vec{j} + 2yz\vec{k}$  prove that  $\operatorname{div}(\operatorname{curl} \vec{f}) = 0$ .  
 (OR)
16. If  $\vec{f} = (2x^2y + yz)\vec{i} + (xy^2 - xz^2)\vec{j} + (pxyz - 2x^2y^2)\vec{k}$  is solinoidal, find the value of p.
17. Find the work done in moving a particle in the  $\vec{F} = 3x^2\vec{i} + (2xz - y)\vec{j} + z\vec{k}$  and c is the straight line from A  $(0, 0, 0)$  to B  $(2, 1, 3)$ .

- (OR)
18. Evaluate  $\iiint_V \nabla \cdot \vec{F} dv$  if  $\vec{F} = x^2 \vec{i} + y^2 \vec{j} + z^2 \vec{k}$  and if v is the volume of the region enclosed by the cube  $0 \leq x, y, z \leq 1$ .
19. Find the sine transform of  $f(t) = e^{-\alpha t}$ ,  $t \geq 0$ ,  $\alpha > 0$  using derivatives.  
(OR)
20. If  $F(s)$  is the fourier transform of  $f(x)$ , then prove that  $F[f(x) \cos ax] = \frac{1}{2} [F(s+a) + F(s-a)]$ .

**SECTION - C ( 5 X 9 = 45 )**

**Answer ALL Questions:**

21. Express  $\frac{\sin 6\theta}{\sin \theta}$  in terms of  $\cos \theta$ .  
(OR)
22. If  $\tan(\theta + i\varphi) = \cos \alpha + i \sin \alpha$  show that  
a)  $\tan 2\varphi = \sin \alpha$    b)  $\cosh 2\varphi = \sec \alpha$ .
23. Sum to infinity the series  

$$\cos \alpha + \frac{1}{2} \cos(\alpha + \beta) + \frac{1.3}{2.4} \cos(\alpha + 2\beta) + \dots$$
  
(OR)
24. Show that  $\log \left\{ \tan \left[ \frac{\pi}{4} + \frac{1}{2} ix \right] \right\} = i \operatorname{tan}^{-1} (\sinh x)$ .
25. If  $\nabla \phi = 2xyz^3 \vec{i} + x^2 z^3 \vec{j} + 3x^2 y z^2 \vec{k}$  find  $\phi(1, -2, 2) = 4$ .  
(OR)
26. Find the directional derivative of  $\varphi = x + xy^2 + yz^3$  at  $(0, 1, 1)$  in the direction of the vector  $2\vec{i} + 2\vec{j} - \vec{k}$ .
27. Evaluate  $\iiint_V \nabla \cdot \vec{A} dv$  if  $\vec{A} = 2x^2 y \vec{i} - y^2 \vec{j} + 4xz^2 \vec{k}$  where V is the region in the first octant bounded by the cylinder  $y^2 + z^2 = 9$  and the plane  $x = 2$ .  
(OR)
28. Verify Stokes' theorem for  $\vec{A} = x^2 \vec{i} + xy \vec{j}$  taken over the square surface g in the xoy plane whose vertices are O (0,0,0), A (a,0,0), B (a,a,0), C (0,a,0) and over its boundary.
29. Find the Fourier transform of  $\frac{1}{\sqrt{|x|}}$ .  
(OR)
30. Find the Fourier sine transform of  $f(x) = \begin{cases} x, & 0 < x < 1 \\ 2-x, & 1 < x < 2 \\ 0, & x > 2 \end{cases}$
- \* \* \* \* \*