S.NO. 338   BATCH: 87–14, 2016, 17   Reg. No.

END OF SEMESTER EXAMINATIONS, APRIL/MAY - 2018
MATHEMATICS PAPER - IV
SUBJECT CODE: 16UBMA04

MAJOR: B.Sc. (Physics)   4   SEMESTER : II
TIME : 3 HOURS   MAX. MARKS: 75

SECTION – A (5 X 2 = 10)

Answer ALL the questions:

1. Define radical axis.

(OR)

2. Show that the circles $x^2 + y^2 - 6x - 9y + 13 = 0$ and $x^2 + y^2 - 2x - 16y = 0$ touch each other.

3. Find the locus of the middle points of a series of parallel chords of an ellipse.

(OR)

4. Prove that the sum of the squares of two conjugate semi diameters of an ellipse is constant.

5. Find the distance between the points (r_1, θ_1) and (r_2, θ_2).

(OR)

6. Write the equation of the normal at the point P whose vectorial angle is α.

7. Prove the intercept form of the equation of a plane.

(OR)

8. Find the equation of the plane passing through the points (x, y, z), (x_2, y_2, z_2), (x_3, y_3, z_3).

9. Write the Characteristics of the equation of a sphere.

(OR)

10. Find the co-ordinates of the centre and radius of the sphere $2x^2 + 2y^2 + 2z^2 - 2x + 4y + 2z - 15 = 0$.

SECTION – B (5 X 4 = 20)

Answer ALL questions:

11. Find the radical centre of the three circles $x^2 + y^2 - x + 3y - 3 = 0$; $x^2 + y^2 - 2x + 2y + 2 = 0$ and $x^2 + y^2 + 2x + 3y - 9 = 0$.

(OR)

12. Obtain the equation of a circle which passes through the point $(1, 2)$ bisects the circumferences of the circle $x^2 + y^2 = 9$ and cuts orthogonally the circle $x^2 + y^2 - 2x + 8y - 7 = 0$.

13. Show that the normals to the ellipse $\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = 1$ at the ends of the chords $\dfrac{lx}{a} + \dfrac{my}{b} - 1 = 0$ and $\dfrac{x}{la} + \dfrac{y}{mb} + 1 = 0$ are concurrent.

(OR)

14. P and Q are extremities of two conjugate diameters of the ellipse $\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = 1$ and S is a focus. Prove that $PQ^2 - (SP - SQ)^2 = 2b^2$.

15. Find the locus of the foot of the perpendiculars drawn from the pole to the tangents to the circle $r = 2a \cos \theta$.

(OR)

16. Find the condition in order that the line $\dfrac{l}{r} = A \cos \theta + B \sin \theta$ may be a tangent to the conic $\dfrac{l}{r} = 1 + e \cos \theta$.

..2..

17. Find the equation of the plane passing through the points $(3,1,2)$, $(3,4,4)$ and perpendicular to the plane $5x + y + 4z = 0$.

(OR)

18. Find the equation of the plane through the point $(1,-2,3)$ and the intersection of the planes $2x - y + 4z = 7$ and $x + 2y - 3z + 8 = 0$.

19. Find the equation of a sphere which touches the sphere $x^2 + y^2 + z^2 - 6x + 2z + 1 = 0$ at the point $(2,-2,1)$ and passes through the origin.

(OR)

20. Find the equation of the sphere having the circle $x^2 + y^2 + z^2 - 2x + 4y - 6z + 7 = 0$, $2x - y + 2z = 5$ for a great circle.

SECTION – C (5 X 9 = 45)

Answer ALL the questions:

21. Find the equation of the two circles passing through the two points $(0,a)$, $(0,-a)$ and touching the straight line $y = mx + c$. If the two circles cut at right angles, show that $c^2 = a^2(2 + m^2)$.

(OR)

22. Find the equation to the circle whose diameter is the common chord of the two circles, $(x-a)^2 + y^2 = a^2$ and $x^2 + (y-b)^2 = b^2$. Find also the length of the common chord.

23. If the normals at the points whose eccentric angles are α, β, γ are concurrent, then show that $\sin(\beta + \gamma) + \sin(\gamma + \alpha) + \sin(\alpha + \beta) = 0$.

(OR)

24. A tangent to the ellipse $\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = 1$ whose centre is C meets the circle $x^2 + y^2 = a^2 + b^2$ at Q and Q^1. Prove that CQ and CQ^1 are conjugate diameters of the ellipse.

25. Prove that the chords of a rectangular hyperbola which subtend a right angle at a focus touch a fixed parabola.

(OR)

26. If the normal at α, β, γ on $\dfrac{l}{r} = 1 + \cos\theta$ meet in the point (ρ, ϕ), show that

   $(1)\, \tan\dfrac{\alpha}{2} + \tan\dfrac{\beta}{2} + \tan\dfrac{\gamma}{2} = 0$

   $(2)\, \alpha + \beta + \gamma = 2n\pi + 2\phi$.

27. Find the equation of the plane passing through the points $(2,-5,-3)$, $(-2,-3,5)$ and $(5,3,-3)$.

(OR)

28. Find the image of the point $(1,-2,3)$ in the plane $2x - 3y + 2z + 3 = 0$.

29. Find the equation of the sphere which passes through the circle $x^2 + y^2 + z^2 - 2x - 4y = 0$, $x + 2y + 3z = 8$ and touches the plane $4x + 3y = 25$.

(OR)

30. Find the condition that the line $\dfrac{x-a}{l} = \dfrac{y-b}{m} = \dfrac{z-c}{n}$ where $l^2 + m^2 + n^2 = 1$ should touch the sphere $x^2 + y^2 + z^2 + 2ux + 2vy + 2wz + d = 0$.

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