

S.NO. 147

BATCH: 87-99, 2003-15 Reg. No.

END OF SEMESTER EXAMINATIONS, APR/MAY - 2018

COMPLEX ANALYSIS - I

SUBJECT CODE: 08UAMA10

12

MAJOR : B.Sc (MATHEMATICS)

SEMESTER : V

TIME : 3 HOURS

MAX. MARKS : 75

SECTION - A (5 X 2 = 10)

Answer ALL questions:

1. Define holomorphic function.

(OR)

2. Show that $u = e^x \cos y$ is harmonic.

3. Define power series.

(OR)

4. Write the Hadamards formula for the radius of convergence.

5. Define conformal mapping.

(OR)

6. Define bilinear transformation.

7. Define simply connected region.

(OR)

8. State Cauchy's theorem for a rectangle.

9. Define integral function.

(OR)

10. State the fundamental theorem of algebra.

SECTION - B (5 X 4 = 20)

Answer ALL questions:

11. Show that the real and imaginary parts of an analytic function satisfy Laplace equation.

(OR)

12. Show that an analytic function with constant modulus is constant.

13. State and prove Abel's theorem.

(OR)

14. State and prove the addition theorem for $\sin z$ and $\cos z$.

15. Find the bilinear transformation which maps $z = \infty, i, 0$ into $w = 0, i, \infty$.

...2...

(OR)

16. What is the region of the w plane into which the rectangular region in the z plane bounded by the lines $x = 0, y = 0, x = 1$ and $y = 2$ is mapped under the transformation $w = z + (2 - i)$?
17. State and prove Gauss mean value theorem.

(OR)

18. Let $f(z)$ be continuous on a contour L of length l and let $|f(z)| \leq M$ on L . Then prove that

$$\left| \int_L f(z) dz \right| \leq Ml.$$

19. Show that every polynomial equation $p(z) = a_0 + a_1 z + a_2 z^2 + \dots + a_n z^n = 0$ where $n \geq 1$ and $a_n \neq 0$ has exactly n roots.

(OR)

20. State and prove Cauchy's inequality.

SECTION - C (5 X 9 = 45)**Answer ALL questions:**

21. State and prove the sufficient condition for $f(z)$ to be analytic.

(OR)

22. If $u = e^x (x \cos y - y \sin y)$, find the analytic function $u + iv$.

23. Prove that the sum function $f(z)$ of the power series $\sum_0^\infty a_n z^n$ represents an analytic function inside its circle of convergence.

(OR)

24. State and prove Cauchy's criterion for uniform convergence.

25. State and prove the necessary conditions for $w = f(z)$ to represent a conformal mapping.

(OR)

26. Discuss the transformation $w = e^z$.

27. State and prove Morera's theorem.

(OR)

28. Derive Cauchy's integral formula for higher derivatives.

29. State and prove the maximum modulus principle.

(OR)

30. State and prove Liouville's theorem.
