S.No.: 190 BATCH: 87-99, 2003-2016 Reg. No.:

44

END OF SEMESTER EXAMINATIONS, NOVEMBER – 2018
COMPLEX ANALYSIS - I
SUBJECT CODE : 08UAMA10

MAJOR : B.Sc.(Mathematics) SEMESTER : V
TIME : 3 HOURS MAX. MARKS: 75

SECTION – A (5 X 2 = 10)

Answer ALL the Questions:

1. Show that $f(z)=z^2$ is differentiable at every point.

[OR]

2. Define Harmonic function.
3. Define the power series.

[OR]

4. State Hadamard's formula for the radius of convergence.
5. Define: (i) Critical points (ii) Ordinary points.

[OR]

6. What do you mean by cross ratio of four points?
7. Define Jordan arc.

[OR]

8. Evaluate $\int_L \frac{dz}{z-a}$ where L represents a circle $|z-a|=r$.
9. State Liouville's Theorem.

[OR]

10. If $a>e$, prove that the equation $e^z = az^n$ has n roots inside the circle $|z|=1$ by using Rouche's theorem.

SECTION – B (5 X 4 = 20)

Answer ALL the Questions:

11. State and prove complex form of CR equations.

[OR]

12. If $f(z)$ and $\overline{f(z)}$ are analytic in a region D. Show that $f(z)$ is constant in that region.
13. State and prove Abel's Theorem on absolute convergence.

[OR]

14. Find the radius of convergence of $\sum \frac{(n!)^2}{2n!}.z^n$.
15. Determine the region D' of the w - plane corresponding to rectangular region D in z - plane bounded by $x=0, y=0, x=1, y=2$ under the transformation $w=z+(1-i)$.

[OR]

16. Prove that every mobius transformation maps circles or straight lines into circles or straight lines.

17. Let $f(z)$ be continuous on a contour L of length l and let $|f(z)| \le m$ on l, then prove that $\left|\int_c f(z)dz\right| \le ml$.

[OR]

18. State and prove Gauss's Mean value theorem.

19. State and prove Cauchy's inequality.

[OR]

20. Prove that all the roots of $z^7 - 5z^3 + 12 = 0$ lie between the circles $|z| = 1$ and $|z| = 2$.

SECTION – C (5 X 9 = 45)

Answer ALL the Questions:

21. Let $f(z) = u(r,\theta) + iv(r,\theta)$ be differentiable at $z = re^{i\theta} \ne 0$ then prove that $\frac{\partial u}{\partial r} = \frac{1}{r}\frac{\partial v}{\partial \theta}$ and $\frac{\partial v}{\partial r} = \frac{-1}{r}\frac{\partial u}{\partial \theta}$. Further prove that $f'(z) = \frac{r}{z}\left(\frac{\partial u}{\partial r} + i\frac{\partial v}{\partial r}\right)$.

[OR]

22. If $f(z)$ is analytic, prove that $\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right)|f(z)|^2 = 4|f'(z)|^2$.

23. Prove that $1 + \frac{a.b}{1.c}z + \frac{a(a+1)(b)(b+1)}{1.2c(c+1)}z^2 + \ldots$ has unit radius of convergence.

[OR]

24. Discuss the exponential function.

25. Find the bilinear transformation which transforms the unit circular disc $|z| \le 1$ onto the unit circular disc $|w| \le 1$.

[OR]

26. By the transformation $w = z^2$, show that the circles $|z - a| = c$, a, c being real in the z - plane correspond to the lamicans in the w - plane.

27. State and prove Morera's theorem.

[OR]

28. Let $f(z)$ be analytic within and on the boundary C of a simply connected region D and Let z_0 be any point within C, then prove that

$$f'(z) = \frac{1}{2\pi i}\int_C \frac{f(z)}{(z - z_0)^2}dz.$$

29. State and prove maximum modulus principle.

[OR]

30. State and prove fundamental theorem of Algebra.

*****