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S.No. 126 BATCH: 87-96, 2003-2015 Reg.No

END OF SEMESTER EXAMINATIONS, NOVEMBER - 2017
REAL ANALYSIS - I
SUBJECT CODE: 08UAMA09

MAJOR: B.Sc.,(Mathematics) **SEMESTER: V**
TIME : 3 HOURS **MAX.MARKS: 75**

SECTION – A (5 X 2 = 10)

Answer ALL the Questions:

1. Given real numbers a *and* b such that $a \le b + \in, \ \forall \in > 0$ then prove that $a \le b$.

(OR)

2. Find Max S and Min S for the interval $S = [0,1]$.
3. Define subsequence and give an example.

(OR)

4. Define countable set and give an example.
5. Find the accumulation point for $\left\{ \frac{1}{n} \mid n = 1,2,3.... \right\}$.

(OR)

6. Define Matrix Space.
7. Prove that the sequence $\{x_n\}$ in $\{s,d\}$ can converge to atmost one point in S.

(OR)

8. Define continuous function on Matrix Space.
9. Define connected set and give an example.

(OR)

10. Find the jump discontinuity for the function defined by $f(n) = \begin{cases} \frac{x}{|x|} & \text{if } x \neq 0 \\ A & \text{if } x = 0 \end{cases}$

SECTION – B (5 X 4 = 20)

Answer ALL the Questions:

11. Prove that for every integer $n > 1$ is either prime or a product of Primes.

(OR)

12. State and prove Additive property of Supremum.
13. Prove that if the function F is one-to-one on its domain, then F is also a function.

(OR)

14. Prove that the set of all real numbers is uncountable.

15. Prove that the union of any collection of open sets is an open set.

(OR)

16. Prove that a set S in R^n is closed if and only if, it contains all its adherent points.

17. Prove that in any matrix space (s,d) every compact subset T is complete.

(OR)

18. Prove the continuity of composite functions.

19. If $f(x)=\frac{1}{x}$ *for* $x>0$ and $A=(0,1)$, then prove that $f(x)$ is continuous on A but not uniformly continuous on A.

(OR)

20. Prove that if f is strictly increasing on a set S in R then f^{-1} exists and is strictly increasing on $f(s)$.

SECTION – C (5 X 9 = 45)

Answer ALL the Questions:

21. Prove that if $e^x = 1+x+\frac{x^2}{2!}+....+\frac{x^n}{n!}+.....$ then the number 'e' is irrational.

(OR)

22. State and prove Cauchy-Schwarz Inequality.

23. Prove that every subset of a countable set is countable.

(OR)

24. Prove that union of countable collection of countable set is also countable.

25. State and prove Bolzano - Weierstrass theorem.

(OR)

26. State and prove Heine - Borel covering theorem.

27. Prove that in Euclidean space R^k every Cauchy sequence is convergent.

(OR)

28. Assume p is an accumulation point of A and $b \in T$. Prove that $\lim_{x\to p} f(x)=b$ if and only if $\lim_{n\to\infty} f(x_n)=b$ for every sequence $\{x_n\}$ of points in $A-\{p\}$ which converges to p.

29. Prove that A metric S is connected if and only if every two-valued function on S is constant.

(OR)

30. State and prove fixed-point theorem for contraction.

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