Reg.No.

S.No. 39 BATCH: 87-96, 2003 - 2016

END OF SEMESTER EXAMINATIONS, APRIL/MAY- 2019
REAL ANALYSIS - II
SUBJECT CODE: 14UAMA13

MAJOR : B.Sc.,(Mathematics)
TIME : 3 HOURS

SEMESTER : VI
MAX.MARKS: 75

SECTION - A (5 X 2 = 10)

Answer ALL Questions:

1. Define right hand derivative at C.

(OR)

2. Define local minimum.
3. Define bounded variation of f on $[a,b]$.

(OR)

4. Define the K^{th} sub interval of P.
5. Define a refinement of P.

(OR)

6. Write the formula for integration by parts.
7. Write the upper and lower Stieltjes sums of f.

(OR)

8. State second fundamental theorem of integral calculus.
9. State Bonnet's theorem.

(OR)

10. How can we interchange the order of integration?

SECTION - B (5 X 4 = 20)

Answer ALL Questions:

11. Let f be defined on an open interval (a,b) and assume that f has a local maximum or local minimum at an interior point c of (a,b). If f has a derivative at c then prove that $f'(c)$ must be 0.

(OR)

12. Assume f' exists and is monotonic on an open interval (a,b). Then prove that f' is continuous on (a,b).
13. If f is monotonic on $[a,b]$, then prove that the set of discontinuities of f is countable.

(OR)

14. Assume that f and g are each of bounded variation on $[a,b]$. Then prove that $V_{f.g} \le AV_f + BV_g$, where $A = Sup\{|g(x)|: x \in [a,b]\}$, $B = Sup\{|f(x)|; x \in [a,b]\}$
15. If $f \in R(\alpha)$ and $f \in R(\beta)$ on $[a,b]$ then $f \in R(c_1\alpha + c_2\beta)$ on $[a,b]$ and prove that $\int_a^b fd(c_1\alpha + c_2\beta) = c_1\int_a^b fd\alpha + c_2\int_a^b fd\beta$.

(OR)

16. State and prove Reduction of Riemann-Stieltjes integral to a finite sum.
17. Assume that $\alpha \nearrow$ on $[a,b]$. Then prove that $\underline{I}(f,\alpha) \le \bar{I}(f,\alpha)$.

(OR)

18. State and prove first mean value theorem for Riemann-Stieltjes integrals.

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19. Let f be continuous at each point (x,y) of a rectangle $Q=\{(x,y);a\le x\le b,c\le y\le d\}$. Assume that α is of bounded variation on $[a,b]$ and let F be the function defined on $[c,d]$ by the equation $F(y)==\int_a^b f(x,y)d\alpha(x)$. Then prove that F is continuous on $[c,d]$.

(OR)

20. Let f be continuous on the rectangle $[a,b]\times[c\times d]$. If $g\in R$ on $[a,b]$ and if $h\in R$ on $[c,d]$ then prove that $\int_a^b\left[\int_c^d g(x)b(y)f(x,y)dy\right]dx=\int_c^d\left[\int_a^b g(x)b(y)f(x,y)dx\right]dy$.

SECTION - C (5 X 9 = 45)

Answer ALL Questions:

21. State and prove chain rule.

(OR)

22. State and prove Generalized mean-value theorem.

23. Let f be of bounded variation on $[a,b]$. Let V be defined on $[a,b], V(x)=V_f(a,x)$ if $a<x\le b$, $V(a)=0$. Then prove that (i) V is a increasing function on $[a,b]$, (ii) $V-f$ is an increasing function on $[a,b]$.

(OR)

24. Let f be defined on $[a,b]$. Then prove that f is of bounded variation on $[a,b]$ iff f can be expressed as the difference of two increasing functions.

25. Let $f\in R(\alpha)$ on $[a,b]$ and let g be a strictly monotonic continuous function defined on an interval S having endpoints c *and* d. Assume that $a=g(c),b=g(d)$. Let h *and* β be the composite functions defined as $h(x)=f[g(x)]$, $\beta(x)=\alpha[g(x)]$ is $x\in S$. Then prove that $h\in R(\beta)$ on S and we have $\int_a^b fd\alpha=\int_c^d hd\beta$. That is $\int_{g(c)}^{g(d)} f(t)d\alpha(t)=\int_c^d f[g(x)]d\{\alpha[g(x)]\}$.

(OR)

26. State and prove Euler's summation formula.

27. Assume that $\alpha\nearrow$ on $[a,b]$. Then prove that the following three statements are equivalent.

(i) $f\in R(\alpha)$ *on* $[a,b]$.

(ii) f satisfies Riemann's condition with respect to α on $[a,b]$.

(iii) $\underline{I}(f,\alpha)=\bar{I}(f,\alpha)$.

(OR)

28. If $f\in R$ and $g\in R$ on $[a,b]$, let $F(x)=\int_a^x f(t)dt$, $G(x)=\int_a^x g(t)dt$ if $x\in[a,b]$. Then Prove that F and G continuous functions of bounded variation on $[a,b]$. Also $f\in R(G)$ *and* $g\in R(F)$ on $[a,b]$ and prove $\int_a^b f(x)g(x)dx=\int_a^b f(x)dG(x)=\int_a^b g(x)dF(x)$.

29. State and prove the change of variable in a Riemann integral.

(OR)

30. Let $Q=\{(x,y);a\le x\le b,c\le y\le d\}$. Assume that α is of bounded variation on $[a,b]$, β is of bounded variation on $[c,d]$ *and* f is continuous on Q. If $(x,y)\in Q$, define $F(y)=\int_a^b f(x,y)d\alpha(x), G(x)=\int_c^d f(x,y)d\beta(y)$. Then prove that $F\in R(\beta)$ *on* $[c,d]$, $G\in R(\alpha)$ on $[a,b]$ and $\int_c^d F(y)d\beta(y)=\int_a^b G(x)d\alpha(x)$.

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