S. NO.: 154 BATCH: 2016, 2017 Reg. No.:

END OF SEMESTER EXAMINATIONS, APRIL / MAY - 2018
NUMBER THEORY
SUBJECT CODE : 14P3MA05/17P3MA05

MAJOR : M.Sc (Mathematics) 3 **SEMESTER : I**
TIME : 3 HOURS **MAX. MARKS: 70**

SECTION – A (5 X 4 = 20)

Answer All the Questions:

1. Prove that for any integer x, $(a,b)=(b,a)=(a,-b)=(a,b+ax)$.

[OR]

2. Prove that the number of primes is infinite.
3. Let p be a prime number. Then prove that $x^2 \equiv 1(\bmod p)$ if and only if $x \equiv \pm 1(\bmod p)$.

[OR]

4. Show that 1387 is composite.
5. Show that the congruence $x^5 \equiv 6(\bmod 101)$ has 5 solutions.

[OR]

6. Prove that in any group G, $ab=ac$ implies $b=c$, and likewise $ba=ca$ implies $b=c$. Also prove that if a is any element of a finite group G with identity element e, then there is a unique smallest positive integer r such that $a^r=e$.
7. By using the Gaussian reciprocity law, find the value of $\left(\frac{-42}{61}\right)$.

[OR]

8. Suppose that p is an odd prime. Let n denote the least positive quadratic non residue modulo p. Then prove that $n<1+\sqrt{p}$.
9. For every positive integer n. prove that $\sigma(n)=\prod_{p^{\alpha}//n}\left(\frac{p^{\alpha+1}-1}{p-1}\right)$.

[OR]

10. Prove that the function $\mu(n)$ is multiplicative and $\sum_{d/n}\mu(d)=\begin{cases}1 & if \quad n=1\\ 0 & if \quad n>1\end{cases}$.

SECTION – B (5 X 10 = 50)

Answer All the Questions:

11. Prove that every integer n greater than 1 can be expressed as a product of primes (with perhaps only one factor). Also prove that the factoring of any integer $n>1$ into primes is unique apart from the order of the prime factors.

[OR]

12. Find $g=(b,c)$ where $b=5033464705$ and $c=3137640337$, and determine x and y such that $bx+cy=g$.
13. State and prove the Chinese Remainder theorem and apply the theorem, find the least positive integer x such that $x \equiv 5(\bmod 7)$, $x \equiv 7(\bmod 11)$, *and* $x \equiv 3(\bmod 13)$.

[OR]

14. Prove that the congruence $f(x)\equiv 0(\bmod p)$ of degree n has atmost n solutions.
15. Prove that if p is a prime then there exist $\phi(\phi(p^2))=(p-1)\phi(p-1)$ primitive roots modulo p^2.

[OR]

16. State and prove lemma of Gauss.
17. If Q is odd and $Q>0$, then prove that $\left(\frac{-1}{Q}\right)=(-1)^{(Q-1)/2}$ and $\left(\frac{2}{Q}\right)=(-1)^{(Q^2-1)/8}$.

[OR]

18. Let p denote a prime. Then prove that the largest exponent e such that $p^e/n!$ is $e=\sum_{i=1}^{\infty}\left[\frac{n}{p^i}\right]$.
19. Let $f(n)$ be a multiplicative function and let $F(n)=\sum_{d/n}f(d)$. Then prove that $F(n)$ is multiplicative.

[OR]

20. Find the number of integers in the set $P=\{1,2,3,......6300\}$ that are divisible by neither 3 nor 4; also the number divisible by none of 3,4 or 5.

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S.NO. 72 BATCH: 2016 Reg. No.

END OF SEMESTER EXAMINATIONS, APRIL/MAY - 2018
REAL ANALYSIS
SUBJECT CODE: 14P3MA02

MAJOR: M.Sc. (Mathematics) SEMESTER : I
TIME : 3 HOURS 4 MAX. MARKS: 70

SECTION – A (5 X 4 = 20)

Answer ALL the Questions:

1. State and prove Cauchy's Condition for Uniform Convergence.

(OR)

2. If $\lim_{n\to\infty} f_n = f$ & $\lim_{n\to\infty} g_n = g$ on $[a,b]$ and $h(x)=\int_a^x f(t)g(t)dt,\ h_n(x)=\int_a^x f_n(t)g_n(t)dt$ for all $x\in[a,b]$, then prove that $h_n \to h$ uniformly on $[a,b]$.
3. Let S be an open connected subset of R^n, let $f:S\to R^m$ be differentiable at each point of S. If $f'(C)=0$ for all $C\in S$, then prove that f is constant on S.

(OR)

4. If f is differentiable at c with total derivative T_c. Then show that the directional derivative $f'(c;u)$ exists for all $u\in R^n$ & $T_c(U)=f'(c;u)$.
5. If $\{E_k\}_{k=1}^{\infty}$ is any countable collection of sets, disjoint or not then prove that $m^*\left(\bigcup_{k=1}^{\infty} E_k\right)\le \sum_{k=1}^{\infty} m^*(E_k)$.

(OR)

6. Prove that the union of a finite collection of measurable sets is measurable.
7. Let f be a bounded measurable function on a set of finite measure E, then prove that f is integrable over E.

(OR)

8. State and prove the monotone convergence theorem.
9. State and prove Jordan's theorem.

(OR)

10. If the function f is Lipschitz on a closed, bounded interval $[a,b]$, then prove that it is absolutely continuous on $[a,b]$.

SECTION – B (5 X 10 = 50)

Answer ALL the Questions:

11. Let α be bounded variation on $[a,b]$. Assume that each term of the sequence $\{f_n\}$ is a realvalued function such that $f_n\in R(\alpha)$ on $[a,b]$, $n=1,2,\ldots$. And $f_n\to f$ uniformly on $[a,b]$, define $g_n(x)=\int_a^x f_n(t)d\alpha(t)$ if $x\in[a,b]$, then we have (i) $f\in R(\alpha)$ on $[a,b]$ (ii) $g_n\to g$ uniformly on $[a,b]$.

(OR)

12. State and prove Dirichelt's test for uniform convergence.
13. State and prove Chain rule on derivatives.

(OR)

14. Assume that one of the partial derivatives $D_1f, D_2f,\ldots D_nf$ exists at c & remaining $n-1$ partial derivatives exists in some n Ball $B(c)$ and are continuous at c. Then prove that f is differentiable at c.
15. Prove that Every interval is measurable.

(OR)

16. Prove that the cantor set c is a closed, uncountable set of measure zero.
17. State and prove Fatou's Lemma.

(OR)

18. If $\{f_n\}\to f$ pointwise on E, then prove that f is integrable over E and $\lim_{n\to\infty}\int_E f_n = \int_E f$.
19. If the function f is monotone on the open interval (a,b), then prove that it is differentiable almost everywhere on (a,b).

(OR)

20. Let the function f be monotone on the closed, bounded interval $[a,b]$. Then prove that f is absolutely continuous on $[a,b] \Leftrightarrow \int_a^b f' = f(b)-f(a)$.

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