S. NO.: 107 BATCH: 2015, 2017 Reg. No.:

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END OF SEMESTER EXAMINATIONS, NOVEMBER - 2017
REAL ANALYSIS
SUBJECT CODE : 17P3MA02

MAJOR : M.Sc (Mathematics) SEMESTER : I
TIME : 3 HOURS MAX. MARKS: 70

SECTION A – (5 X 4 = 20)

Answer All the Questions:

1. If P^* is a refinement of P. Prove that $L(P,f,\alpha) \le L(P^*,f,\alpha)$ and $L(P,f,\alpha) \le L(P^*,f,\alpha)$.

(OR)

2. State and prove the fundamental theorem of calculus.
3. Prove that the sequence of functions $\{f_n\}$, defined on E, converges uniformly on E if and only if for every $\varepsilon > 0$ there exists an integer N such that $m \ge N, n \ge N$ implies $|f_n(x) - f_m(x)| \le \varepsilon$.

(OR)

4. If K is a compact metric space, if $f_n \in \mathscr{C}(K)$ for n =1,2,3,…and if $\{f_n\}$ converges uniformly on K. prove that $\{f_n\}$ is equicontinuous on K.
5. Suppose $\sum c_n$ converges. Put $f(x) = \sum_{n=0}^{\infty} c_n x^n \ (-1 \le x \le 1)$. Prove that $\lim_{n \to \infty} f(x) = \sum_{n=0}^{\infty} c_n$.

(OR)

6. Suppose $a_0, \dots, a_n$ are complex numbers, $n \ge 1, a_n \ne 0, P(z) = \sum_0^n a_k z^k$. Prove that $P(z) = 0$ for some complex number z.
7. Let Ω be the set of all invertible linear operator on R^n. If $A \in \Omega, B \in L(R^n)$, and $\|B - A\| . \|A^{-1}\| < 1$. Prove that $B \in \Omega$.

(OR)

8. Suppose E is on open set in R^n, f maps E into R^m, f is differentiable at $x_0 \in E$, g maps an open set containing $f(E)$ into R^k, and g is differentiable at $f(x_0)$. Prove that the mapping f of E into R^k defined by $F(x) = g(f(x))$ is differentiable at x_0, and $F'(x_0) = g'(f(x_0)) f'(x_0)$.
9. If $[A]$ and $[B]$ are n by n matrices, prove that $\det([B][A]) = \det[B] \ \det[B]$.

(OR)

10. Suppose f is defined on an open set $E \subset R^2$, suppose that $D_1 f, D_{21} f$ and $D_2 f$ exist at every point of E, $D_{21} f$ exists at every point of E, and $D_{21} f$ is continuous at some point $(a,b) \in E$. Show that $D_{21} f$ exist at (a,b) and $(D_{12} f)(a,b) = (D_{21} f)(a,b)$.

SECTION B – (5 X 10 = 50)

Answer All the Questions:

11. Assume α increases monotonically and $\alpha' \in \mathfrak{R}$ on $[a,b]$. Let f be a bounded real function on $[a,b]$. Prove that $f \in \mathfrak{R}$ if and only if $f\alpha' \in \mathfrak{R}$ and in that case $\int_a^b f d\alpha = \int_a^b f(x) \alpha'(x) dx$.

(OR)

12. If γ' is continuous on $[a,b]$, prove that γ is rectifiable, and $\wedge(\gamma) = \int_a^b |\gamma'(t)|\,dt$.

13. Suppose $\{f_n\}$ is a sequence of functions, differentiable on $[a,b]$ and such that $\{f_n(x_0)\}$ converges for some point x_0 on $[a,b]$. If $\{f_n'\}$ converges uniformly on $[a,b]$. Prove that $\{f_n\}$ converges uniformly on $[a,b]$, to a function *f*, and $f'(x) = \lim_{n\to\infty} f_n'(x)$, $(a \le x \le b)$.

(OR)

14. If K is compact, if $f_n \in \mathscr{C}(K)$ for n = 1,2,3,... and if $\{f_n\}$ is pointwise bounded and equicontinuous on K, prove that $\{f_n\}$ is uniformly bounded on K and $\{f_n\}$ contains a uniformly convergent subsequence.

15. State and prove Taylor's theorem.

(OR)

16. State and prove Parseval's theorem.

17. Let *r* be positive integer. If a vector space X is spanned by a set of *r* vectors. Prove that $\dim X \le r$.

(OR)

18. State and prove inverse function theorem.

19. State and prove implicit function theorem.

(OR)

20. State and prove the rank theorem.

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