

END OF SEMESTER EXAMINATIONS, NOVEMBER – 2017
 MATHEMATICAL PHYSICS
 SUBJECT CODE: 09P3PH02

MAJOR: M.Sc.,(Physics)
 TIME : 3 HOURS

SEMESTER : I
 MAX.MARKS: 70

Section-A (10 × 1 = 10)

Answer ALL Questions

- If $\text{div } \vec{V} = 0$ then \vec{V} becomes
 - incompressible field
 - lamellar vector field
 - solenoidal vector field
 - both (a) and (c) are correct.
- Laplace transform of K (constant) is
 - $\frac{1}{s}$
 - $\frac{1}{s^2}$
 - $\frac{s}{k}$
 - $\frac{k}{s}$
- The only function among the following that is analytic, is :
 - $f(z) = R(iz)$
 - $f(z) = \text{Im}(z)$
 - $f(z) = \bar{z}$
 - $f(z) = \sin z$
- $\int_{-1}^1 [P_n(x)]^2 dx =$
 - 0
 - 1
 - $\frac{2}{2n+1}$
 - $\frac{4}{4n+1}$
- Euler's formula for solving first order differential equation is
 - $y_{n+1} = y_n + hf(x_n, y_n)$
 - $y_{n+1} = y_n + \frac{h}{2} f(x_n, y_n)$
 - $y_{n+1} = y_n + hf(x_0, y_0)$
 - $y_{n+1} = y_0 + hf(x_n, y_n)$
- Define the condition for a set of n vectors becomes orthonormal set.
- State the sufficient condition for the existence of Laplace transform
- Enunciate Cauchy-Riemann differential equation.
- What is the Rodrigue's formula for Legendre polynomials?
- Write the condition for one real root exists between 'a' and 'b' in Bisection method.

Section-B (5 × 4 = 20)

Answer ALL Questions

- (a) State and prove Green's theorem in a plane. (OR)
 (b) Express the Laplacian operator in terms of spherical polar and cylindrical co-ordinates.
- (a) State Dirichlet's theorem? List out the conditions imposed by Dirichlet's theorem on $f(x)$. (OR)
 (b) Enumerate the uses of Fourier series.
- (a) State and prove Cauchy's integral theorem. (OR)
 (b) Evaluate the integral $\oint_C \frac{dz}{z^2+z}$; where C is a circle defined by $|z| = |R|$.
- (a) Prove that $J_{-1/2}(x) = \sqrt{\frac{2}{\pi x}} \cos x$. (OR)
 (b) Show that $x J'_n = n J_n - x J_{n+1}$.
- (a) Using Newton's method, find the root between 0 and 1 of $x^3 = 6x - 4$ correct to 5 decimal places. (OR)
 (b) Evaluate $\int_0^1 e^x dx$ by Simpson's one-third rule correct to five decimal places, by proper choice of h.

Section-C (5 × 8 = 40)

Answer ALL Questions

16. (a) Obtain set of orthonormal vectors for a set of non-orthonormal but linearly independent vectors using Schmidt's orthogonalization method. (OR)
 (b) State and prove Gauss' divergence theorem. Hence deduce Green's theorem.
17. (a) Find the series of sines and cosines of multiple of x which represents $f(x)$ in the interval $-\pi < x < \pi$ where

$$f(x) = \begin{cases} 0 & \text{when } -\pi < x < \pi \\ \frac{\pi x}{4} & \text{when } 0 < x \leq \pi \end{cases}$$

and hence deduce $\frac{\pi^2}{8} = 1 + \frac{1}{3^2} + \frac{1}{5^2} + \dots$ (OR)

(b) Describe various properties of Inverse Laplace Transform along with their proof.

18. (a) Express Cauchy Residue theorem and give its proof. (OR)
 (b) Apply calculus of residues to show that $\int_0^{2\pi} \frac{d\theta}{a+b \cos \theta} = \frac{2\pi}{\sqrt{a^2-b^2}}$, $a > b > 0$.
19. (a) Obtain Rodrigue's formula for Legendre Polynomials. (OR)
 (b) Prove the following recurrence formula for $P_n(x)$
- $$(1) nP_n = (2n-1)xP_{n-1} - (n-1)P_{n-1}$$
- $$(2) nP_n = xP'_n - P'_{n-1}$$
20. (a) Solve the system by Gauss-Elimination method $2x + 3y - z = 5$; $4x + 4y - 3z = 3$ and $2x - 3y + 2z = 2$. (OR)
 (b) Compute $y(0.3)$ given $\frac{dy}{dx} + y + xy^2 = 0$, $y(0) = 1$ by taking $h = 0.1$ using R.K method of fourth order (correct to four decimals)
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