

END OF SEMESTER EXAMINATIONS, APRIL / MAY - 2017  
 MATHEMATICAL STATISTICS  
 SUBJECT CODE: 14P3MA06

MAJOR: M.Sc MATHS  
 TIME : 3 HOURS

SEMESTER : II  
 MAX. MARKS : 70

SECTION - A (5 X 4 = 20)

Answer all the Questions:

1. Let  $\{A_n\}$ ,  $n = 1, 2, 3, \dots$  be a non decreasing sequence of events and let A be their alternative. Then prove that  $P(A) = \lim_{n \rightarrow \infty} P(A_n)$

(OR)

2. We have two urns. There are 3 white and 2 black balls in the first urn and 1 white and 4 black balls in the second. From an urn chosen at random we select one ball at random. What is the probability of obtaining white ball if the probability of selecting each of the urns equals 0.5?
3. Define uniform distribution and find its characteristic function and its properties.

(OR)

4. If  $X_1$  and  $X_2$  are two independent Poisson random variables with parameters  $\lambda_1$  and  $\lambda_2$  then find the characteristic function of the random variable  $Z = X_1 - X_2$ .
5. Let the random variable  $X_n$  has a binomial distribution given by

$$P(X_n = r) = \frac{n!}{r!(n-r)!} p^r (1-p)^{n-r}, \quad r = 0, 1, 2, \dots, n$$

If there holds, the relation  $p = \frac{\lambda}{n}$ ,  $n = 1, 2, 3, \dots$  where  $\lambda > 0$  is constant then prove that

$$\lim_{n \rightarrow \infty} P(X_n = r) = \frac{\lambda^r}{r!} e^{-\lambda}$$

(OR)

6. If the equation  $E(x) = \nu \frac{p}{q} = c$ , where c is a positive constant, is satisfied for every  $\nu$ , then prove that the probability function of the negative binomial distribution tends to the corresponding function of the Poisson distribution as  $\nu \rightarrow \infty$ .
7. Explain about the distribution of the  $\phi$  arithmetic mean of independent normally distributed random variables.

(OR)

8. Compute the probability that  $\chi^2$  will exceed or equal 10 for an independent random variables  $X_k$  ( $k = 1, 2, \dots, 8$ ) have same normal distribution  $N(0, 2)$  with statistic

$$\chi^2 = \sum_{k=1}^8 X_k^2 \text{ has eight degrees of freedom.}$$

9. Define asymptotically most efficient estimate and give an example.

(OR)

10. Write the property of the most efficient estimate.

...2...

SECTION - B (5 X 10 = 50)Answer all the Questions:

11. Determine the distribution functions of sum, difference, product and ratio of two independent random variables X and Y.

(OR)

12. Prove that the equality  $p^2 = 1$  is a necessary and sufficient condition for the relation

$$p(Y = aX + b) = 1, \text{ to hold.}$$

13. What are Semi-invariants and express the semi-invariants in terms of the moments or moments in terms of the semi-invariants

(OR)

14. Let  $F(x)$  and  $\phi(t)$  denote respectively the distribution function and the characteristic function of the random variable x. If  $a+h$  and  $a-h$  ( $h>0$ ) are continuity points of the

$$\text{distribution function } F(x) \text{ then prove that } F(a+h) - F(a-h) = \lim_{T \rightarrow \infty} \frac{1}{\pi} \int_{-T}^T \frac{\sinh t}{t} e^{-ita} \phi(t) dt$$

15. Obtain  $m_1, m_2$  and  $\mu_2$  for uniform and Normal distributions.

(OR)

16. State and Prove Levy-Cramer theorem.

17. Find the common distribution of the statistic  $\bar{x}$  and s.

(OR)

18. Derive Fisher's z distribution.

19. State and prove Rao-Cramer inequality.

(OR)

20. Briefly explain the characteristic of a parametric significance test (If one parameter Q is unknown)

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