S.No.: 102 BATCH: 2017 Reg. No.:

END OF SEMESTER EXAMINATIONS, NOVEMBER – 2018
TOPOLOGY
SUBJECT CODE : 17P3MA10

MAJOR : M.Sc.(Mathematics) SEMESTER : II
TIME : 3 HOURS MAX. MARKS: 70

SECTION – A (5 X 4 = 20)

Answer ALL the Questions:

1. Let Y be a subspace of X. If U is open in Y and Y is open in X. Show that U is open in X.

[OR]

2. Prove that every finite point set in a Hausdroff space X is closed.
3. State and prove the uniform limit theorem.

[OR]

4. Show that the union of a collection of connected subspaces of X that have a point in common is connected.
5. Show that every closed subspace of a compact space is compact.

[OR]

6. Prove that every compact Hausdorff space is normal.
7. Prove that the product of completely regular space is completely regular.

[OR]

8. Let X be a completely regular space. Prove that there exist a compactification Y of X having the property that every bounded continuous map $f : X \to R$ extends uniquely to a continuous map of Y into R.
9. Let X be a normal space; Let A be a closed G_s set in X. Prove that there is a continuous function $f : X \to [0,1]$ such that $f(x)=0$ for $x \in A$ and $f(x)>0$ for $x \notin A$.

[OR]

10. Let X be a Para compact Hausdorff space; Let $\{U_\alpha\}_{\alpha \in J}$ be an indexed open covering of X. Prove that there exists a partition of unity on X denoted by $\{U_\alpha\}$.

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SECTION – B (5 X 10 = 50)

Answer ALL the Questions:

11. Prove that in any topological space X, the following conditions hold.
    i. ϕ and X are closed.
    ii. Arbitrary intersections of closed sets are closed.
    iii. Finite unions of closed sets are closed.

[OR]

12. Let X,Y and Z be topological spaces. Prove the following statements.
    i. If $f : X \to Y$ maps all of X into the single point Y_0 of Y then f is continuous.
    ii. If A is a subspace of X; the inclusion function $j = A \to X$ is continuous.
    iii. If $f : X \to Y$ and $g : Y \to Z$ are continuous then the map $g \circ f : X \to Z$ is continuous.

13. Let X be a metric space with metric d. Define $\bar{d} : XXX \to R$ by the equation $\bar{d}(x, y) = \min\{d(x, y), 1\}$ prove that $\bar{d}$ is a metric that induces the topology of X.

[OR]

14. Prove that a finite Cartesian product of connected space is connected.

15. Prove that the product of finitely many compact spaces is compact.

[OR]

16. Prove that every regular space with a countable basis is normal.

17. State and prove Urysohn's metrization theorem.

[OR]

18. State and prove the Tietze extension theorem.

19. State and prove Nagata – smirnor metrization theorem.

[OR]

20. Let X be regular, then prove that the following conditions are equivalent: Every open covering of X has a refinement that is:
    i. An open covering of X and countably locally finite.
    ii. A Covering of X and locally finite.
    iii. A closed covering of X and locally finite.
    iv. An open covering of X and locally finite.

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