S. NO.: 71 BATCH: 2014-2016 Reg. No.:

END OF SEMESTER EXAMINATIONS, APRIL / MAY - 2017
GRAPH THEORY
SUBJECT CODE : 14P3MA07

MAJOR : M.Sc. (MATHS) SEMESTER : II
TIME : 3 HOURS MAX. MARKS: 70

SECTION A – (5 X 4 = 20)

Answer All the Questions:

1. Explain about Konigsborg Bridge problem.

[OR]

2. Prove that, the number of vertices of odd degree in a graph is always even.
3. Prove that, there is a one and only one path between every pair of vertices in a tree, T.

[OR]

4. In a graph G there is one and only one path between every pair of vertices, then prove that G is a tree.
5. Prove that, two graphs G_1 and G_2 are isomorphic if and only if their incident matrixes $A(G_1)$ and $A(G_2)$ differ only by permutations of rows and columns.

[OR]

6. If $A(G)$ is an incidence matrix of a connected graph G with n vertices, then prove that the rank of $A(G)$ is $n-1$.
7. Prove that $d^-(v)=d^+(v)$ for every vertex v in G.

[OR]

8. Explain Simple Digraphs, Asymmetric Digraphs and Symmetric Digraphs.
9. Prove that, the number of simple, labled graphs of n vertices is $2^{\frac{n(n-1)}{2}}$.

[OR]

10. Prove that, there are n^{n-2} labled trees with n-vertics $(n \geq 2)$.

SECTION B – (5 X 10 = 50)

Answer All the Questions:

11. Prove that, a simple graph with n vertices and k components can have at most $\frac{(n-k)(n-k+1)}{2}$ edges.

[OR]

12. Show that the maximum number of edges in a simple graph with n vertices is $\frac{n(n-1)}{2}$.
13. Prove that a connected graph G is a tree if and only if adding an edge between any two vertices in G creates exactly one circuit.

[OR]

14. Prove that, every circuit has an even number of edges in common with any cut – set.
15. Prove that, the rank of cut-set matrix $C(G)$ is equal to the rank of incidence matrix $A(G)$, which is equals the rank of graph G.

[OR]

16. If the edges of a connected graph are arranged in the same order for the columns of the incidence matrix A and the path matrix $P(x,y)$, then prove that the product $(\text{mod}\,2)\,A\cdot P^T(x,y)=M$, where the matrix M has 1's in two rows x and y, and the rest of $n-2$ rows are all 0's.
17. Prove that, an arborescence is a tree in which every vertex other than the root has an in – degree of exactly one.

[OR]

18. Prove that, an arborescence there is a directed path from the root R to every other vertex iff a circuit less digraph G is an arborescence if there is a vertex v in G such that every other vertex is accessible from v, and v is not accessible from any other vertex.
19. Prove that a vertex v appears in sequence $(b_1,b_2,\ldots b_{n-2})$ m times iff degree of $v=m-1$.

[OR]

20. Explain composition of permutations and permutation Group.

****