S.No. 160 BATCH: 2014 – 2016 Reg.No.

END OF SEMESTER EXAMINATIONS, NOVEMBER - 2017
GRAPH THEORY
SUBJECT CODE: 14P3MA07

MAJOR: M.Sc., (Mathematics) **SEMESTER: II**
TIME : 3 HOURS **MAX.MARKS: 70**

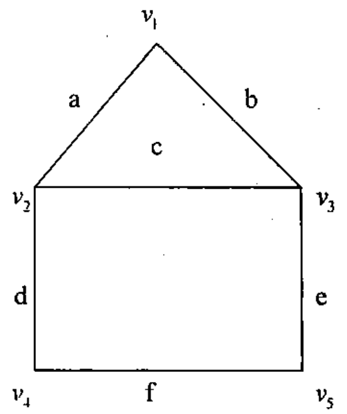
SECTION – A (5 X 4 = 20)

Answer ALL the Questions:

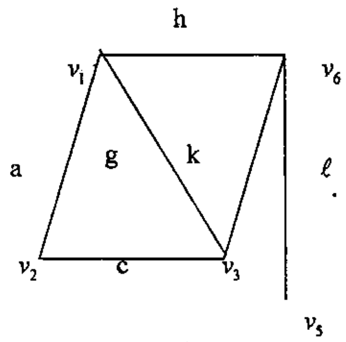
1. Define a graph and regular graph. Give example of a regular graph of degree 2 & 3.

(OR)

2. Find $G_1 \cap G_2$, $G_1 \cup G_2$ and $G_1 \oplus G_2$, from the graphs G_1 and G_2 given below:



G_1



G_2

3. Prove that every connected graph has atleast one spanning tree.

(OR)

4. Define edge connectivity, vertex connectivity of a graph with example.
5. If B is a circuit matrix of a connected graph G with e edges and n vertices, then prove that the rank of $B = e - n + 1$.

(OR)

6. Let $A(G)$ be an incidence matrix of a connected graph G with n vertices. Prove that an $(n-1)$ *by* $(n-1)$ sub matrix of $A(G)$ is non-singular if and only if the $(n-1)$ edges corresponding to the $(n-1)$ columns of this matrix constitute a spanning tree in G.
7. Define simple digraph, symmetric digraph and isograph with example.

(OR)

8. Define arborescene. Show that an arborescene is a tree in which every vertex other than the root has an in-degree of exactly one.

..2.,

9. What are the two categories of graph enumeration problems?. Prove that the number of simple, labeled graphs of n vertices is $2^{\frac{n(n-1)}{2}}$.

(OR)

10. Draw all rooted labeled trees of one, two, and three vertices.

SECTION – B (5 X 10 = 50)

Answer ALL the Questions:

11. Define connected and disconnected graph with example. Prove that a simple graph with n vertices and k components can have atmost $\frac{(n-k)(n-k+1)}{2}$ edges.

(OR)

12. Prove that a given connected graph G is Euler graph if and only if all vertices of G are of even degree.

13. What is distance and center in a tree? Show that a tree has a center consisting of either one vertex or two adjacent vertices.

(OR)

14. Prove that every circuit has an even number of edges in common with any cut-set.

15. State and prove the theorem relating A-incidence matrix of G, B-circuit matrix of G.

(OR)

16. Define Adjacency matrix of a graph with example. What are the observation that can be made about the adjacency matrix of a graph?

17. Explain Teleprinter's Problem.

(OR)

18. Prove that the determinant of every square submatrix of A, the incidence matrix of a digraph, is 1, -1, or 0.

19. Prove that there are n^{n-2} labeled trees with n vertices $(n \geq 2)$.

(OR)

20. Derive cycle index of the full symmetric group of degree four.

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