S. NO.: 59

BATCH: 2017

Reg. No.:

END OF SEMESTER EXAMINATIONS, APRIL / MAY - 2018 MATHEMATICAL PHYSICS - II

SUBJECT CODE: 17P3PH05

MAJOR: M.Sc (Physics)
TIME: 3 HOURS

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SEMESTER : II MAX. MARKS: 70

SECTION - A $(10 \times 1 = 10)$

Answer All the Questions:

Choose the best Answer:

- 1. The product of a matrix and its transpose matrix is a unit matrix, then the matrix is called
 - a. a upper triangular matrix

b. a lower triangular matrix

c. an orthogonal matrix

- d. an unitary matrix
- 2. The outer product of two mixed tensors A_i and B_i^k is
 - a. A scalar

b. a tensor of rank 2

c. a tensor of rank 4

- d. a tensor of rank 3
- 3. The solution of the equation $\frac{d^2y}{dx^2} 8\frac{dy}{dx} + 15y = 0$ is

a. $y = C_1 e^{2x} + C_2 e^{5x}$

b.
$$y = C_1 e^{5x} + C_2 e^{4x}$$

c. $y = C_1 e^{5x} + C_2 e^x$

d.
$$y = C_1 e^{3x} + C_2 e^{5x}$$

- 4. The value of Hermite polynomial $H_2(x)$ is
 - a. 4x-2
- b. 4x-12
- c. 2x
- d. $4x^2 2$
- 5. If any two elements A and B of a group commute, then the group is said to be
 - a. Cyclic group
- b. abelian group
- c. non-Abelian group
- d. permutation group
- 6. Define Hermitian matrix.
- 7. Define a contravariant tensor of order two.
- 8. Find the solution of the equation $\frac{d^2y}{dx^2} 6\frac{dy}{dx} + 9y = 0.$
- 9. Define Gamma function.
- 10. Define a subgroup.

SECTION - B $(5 \times 4 = 20)$

Answer All the Questions:

11. a) Define an orthogonal matrix. Show that $A = \begin{bmatrix} \cos \theta & 0 & \sin \theta \\ 0 & 1 & 0 \\ -\sin \theta & 0 & \cos \theta \end{bmatrix}$ is orthogonal.

[OR]

- b) State any four theorems on eigen values and eigen vectors.
- 12. a) Define covariant and mixed tensor of order three.

[OR]

- b) Define a tensor. Show that Kronecker delta is a mixed tensor of order two.
- 13. a) Write the solution of the auxillary equation when the roots are
 - i) real and different
 - ii) real and equal and
 - iii) imaginary.

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b) Solve: $(D^2 + 4)y = \cos 2x$.

14. a) Derive the generating function of Hermite polynomials.

[OR]

- b) Establish the relation between Beta and Gamma functions.
- 15. a) State and prove Schur's lemma I in group theory.

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- b) Explain i) Isomorphism and Homomorphism.
 - ii) Reducible and Irreducible representations in group theory.

SECTION - C $(5 \times 8 = 40)$

Answer All the Questions:

16. a) Find the characteristic equation of the matrix $A = \begin{pmatrix} 1 & 2 & 3 \\ 2 & -1 & 4 \\ 3 & 1 & 1 \end{pmatrix}$ and verify the Cayley –

Hamilton theorem for it.

[OR]

b) Diagonalise the following matrix:
$$A = \begin{pmatrix} \cos \theta & -\sin \theta & 0 \\ \sin \theta & \cos \theta & 0 \\ 0 & 0 & 1 \end{pmatrix}$$
.

- 17. a) Explain:
 - i) Symmetric and antisymmetric tensors.
 - ii) Addition of two tensors.
 - iii) Outer product of two tensors.
 - iv) Inner product of two tensors.

[OR]

- b) State and prove Quotient law in tensor analysis.
- 18. a) A body executes damped forced vibrations that are governed by the differential equation $\frac{d^2x}{dt^2} + 2k\frac{dx}{dt} + b^2x = e^{-kt}\sin\omega t$. Where all the symbols have their usual meaning. Solve the above differential equation for both the cases, where $\omega^2 \neq (b^2 k^2)$ and $\omega^2 = (b^2 k^2)$.

b) Solve:
$$(D^2 - 7D + 10)y = e^{2x} + e^{5x}$$
.

19. a) Solve the Laguerre's differential equation $x \frac{d^2y}{dx^2} + (1-x)\frac{dy}{dx} + ny = 0$.

[OR]

b) Prove the orthogonally relation of Hermite polynomial in the form

$$\int_{0}^{\infty} e^{-x^{2}} H_{m}(x) H_{n}(x) dx = 2^{n} n! \pi^{\frac{1}{2}} \delta_{mn}.$$

20. a) State and prove the great orthogonally theorem in group theory.

[OR]

b) Obtain multiplication table for the group of symmetry operations of a square (C_{4U} group).
