S.NO. 259 BATCH: 2017 Reg. No.

END OF SEMESTER EXAMINATIONS, APRIL / MAY-2018
TOPOLOGY
SUBJECT CODE: 17P3MA10

MAJOR: M.Sc (Mathematics) **SEMESTER : II**
TIME : 3 HOURS **MAX. MARKS: 70**

SECTION – A (5 X4 = 20)

Answer All the Questions:

1. "Let X be a set and B be a basis for a Topology τ on X then τ equals the collection of all union of elements of B"-Prove.

(OR)

2. Show that let y be a subspace of X then a set A is closed in Y if it equals the intersection of a closed set of X with Y.
3. State and prove sequence lemma.

(OR)

4. State and prove intermediate value theorem.
5. Prove that if Y be a subspace of X then y is compact if and only if every covering of Y by sets open in X contains a finite sub collection covering Y.

(OR)

6. Show that if a subspace of a Hausdorff space of Hausdorff then a product of Hausdorff spaces is Hausdorff.
7. State Imbedding theorem define separate points from closed sets.

(OR)

8. Prove that if $A \subset X$ and $f: A \to Z$ be a continuous map of A into the Hausdorff Space Z then there is at most one extension of f to a continuous function g: $\overline{A} \to Z$.
9. Let A be a locally finite collection of subsets of X then $\overline{\bigcup_{A\in A} A} = \bigcup_{A\in A} \overline{A}$ -Prove.

(OR)

10. Show that very paracompact Hausdorff space X is normal.

SECTION – B (5 X 10 = 50)

Answer All the Questions:

11. Prove that if X be an ordered set in the order topology and Y be a subset of X that is convex in X then the order topology on Y is the same as the topology Y inherits as a subspace of X.

(OR)

12. State and prove a) The pasting lemma b) Maps into products
13. The topologies on $\mathbb{R}^n$ induced by the euclidean metric d and the square metric ρ are the same as the product topology of $\mathbb{R}^n$ -Prove.

(OR)

14. Show that if L is a linear continum in the order topology then L is connected, and so are intervals and rays in L.
15. Prove that if X be a non empty compact Hausdorff space and X has no isolated points then X is uncountable.

(OR)

16. Every well – ordered set X is normal in the order topology - Prove.
17. State and prove Urysohn lemma.

(OR)

18. State and prove Tychonoff theorem.
19. State and prove Nagata-Smirnov metrization theorem

(OR)

20. Show that space X is metrizable if and only if it is a paracompact Hausdorff space that is locally metrizable.

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Reg.No.

S.No. 94 BATCH: 2014 - 2016

END OF SEMESTER EXAMINATIONS, APRIL / MAY -2018
TOPOLOGY AND FUNCTIONAL ANALYSIS
SUBJECT CODE: 14P3MA11

MAJOR: M.Sc., (Mathematics) 10 **SEMESTER: III**
TIME : 3 HOURS **MAX.MARKS: 70**

SECTION – A (5 X 4 = 20)

Answer ALL Questions:

1. Prove that the interior $Int(E)$ of a set E in a space X is the largest open set of X contained in E.

(OR)

2. Prove that E of a space X is open in X if and only if the inclusion map $i: E \subset X$ is open.
3. Show that every regular Frechet space is a Hausdorff space.

(OR)

4. Prove that every closed set K in a compact space X is compact.
5. Show that every continuous image of a connected set is connected.

(OR)

6. Prove that every contractible space is pathwise connected.
7. Show that ℓ_p^n is a Banach Space.

(OR)

8. Prove that $Lp^* = Lq$; where $\frac{1}{p} + \frac{1}{q} = 1$.
9. Prove that every non zero Hilbert space contains a complete Orthonormal set.

(OR)

10. If m is a closed linear subspaces of a Hilbert space H, then prove that $H = M \oplus M^{\perp}$.

SECTION – B (5 X 10 = 50)

Answer ALL Questions:

11. Prove that a set U of a space X is open iff U contains a neighborhood of each of its points.

(OR)

12. Prove that set E in a space X is dense in X iff the topology of the space X has a basis such that every non empty basic open set meets E.
13. If E is a retract of a Hausdorff space X, then prove that E is closed set in E.

(OR)

14. State and prove Tychonoff's theorem.
15. Prove that the topological product of an arbitrary family of connected spaces is connected.

(OR)

16. Show that space X is locally pathwise connected of a point p iff every neighborhood of p contains a pathwise connected neighborhood of p
17. State and prove the Hahn Banach theorem.

(OR)

18. State and prove the open mapping theorem.
19. If x and y are two vectors in a Hilbert space then prove that $|(x,y)| \le \|x\| \, \|y\|$.

(OR)

20. State and prove Bessels inequality.

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