S.NO. 234 BATCH: 2014 – 2016 Reg. No.

END OF SEMESTER EXAMINATIONS, APRIL / MAY-2018
MATHEMATICAL METHODS
SUBJECT CODE: 14P3MA13

MAJOR: M.Sc. Mathematics | **SEMESTER : III**
TIME : 3 HOURS **MAX. MARKS: 70**

SECTION – A (5 X 4 = 20)

Answer ALL the Questions:

1. Find the Cosine transform of the function f(x), if

$$f(x)=\begin{cases}\cos x, & 0<x<a\\ 0, & x>a\end{cases}$$

(OR)

2. Find the finite Cosine transform of $\left(1-\frac{x}{\pi}\right)^2$.

3. Find the Hankel transform of $f(x)=\begin{cases}1, & 0<x<a, \quad n=0\\ 0, & x>a, \quad n=0\end{cases}$

(OR)

4. Show that $\int_0^a rJ_0(P_r)dr=\frac{a}{p}J_1(ap)$.

5. Solve the homogeneous Fredholm integral equation $g(s)=\lambda\int_0^1 e^s e^t g(t)dt$.

(OR)

6. State and prove Fredholm theorem.

7. Reduce the Initial Value problem $y''(s)+\lambda y(s)=F(s)$ y(0)=1, $y'(0)=0$ to a Volterra integral equation.

(OR)

8. Transform the following boundary value problem to integral equations
$y''+sy=1, y(0)=0, y(1)=0$.

9. On what Curves can the functional $v[y(x)]=\int_0^{\frac{\pi}{2}}(y'^2-y^2)dx$, $y(0)=0$, $y\left(\frac{\pi}{2}\right)=1$ be extremized.

(OR)

10. Find the extremal of the functional $v[y(x)]=\int_0^1\left(1+y''^2\right)dx$ with

$y(0)=0$, $y'(0)=1$, $y(1)=1$, $y'(1)=1$.

..2...

SECTION – B (5 X 10 = 50)

Answer ALL the Questions:

11. Find the Fourier transform of $F(x)=\begin{cases}1-x^2, & |x|\le 1\\ 0, & |x|>1\end{cases}$ and hence evaluate

$$\int_0^{\infty}\left(\frac{x\cos x-\sin x}{x^2}\right)\cos\frac{x}{2}\,dx$$

(OR)

12. Solve $\frac{\partial u}{\partial t}=\frac{\partial^2 U}{\partial x^2}$, $x>0, t>0$ Subject to the Conditions

(i) $U=0$, *when* $x=0$, $t>0$

(ii) $U=\begin{cases}1, & 0<x<1\\ 0, & x\ge 1\end{cases}$ *when* $t=0$ and

(iii) $U(x,t)$ is bounded.

13. Find the Hankel transform of (a^2-r^2), if $rJ_0(p_r)$ is the kernel of the transform.

(OR)

14. Find the Hankel transform of $f(x)=\begin{cases}a^2-x^2, & 0<x<a, \quad n=0\\ 0, & x>a, \quad n=0\end{cases}$

15. Solve the Fredholm integral equation of the second kind $g(s)=S+\lambda\int_0^1\left(st^2+s^2t\right)g(t)dt$

(OR)

16. Show that the integral equation $g(s)=f(s)+\frac{1}{\pi}\int_0^{2\pi}\left(\sin(s+t)\right)g(t)dt$ Possesses no solution for f(s) =s, but that it possesses infinitely many solutions when f(s)=1.

17. Solve the integral equation $s=\int_0^S\frac{g(t)dt}{(s-t)^{1/2}}$.

(OR)

18. Explain Boundary value problem.

19. Find the extremal of the functional $v\left[y(x),z(x)\right]=\int_0^{\pi/2}\left[y'^2+z'^2+2yz\right]dx$, y(0)=0, $y(\frac{\pi}{2})=1$, z(0)=0, $z(\frac{\pi}{2})=-1$.

(OR)

20. Derive Euler's equation.

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