S. NO: 265 BATCH: 2013 - 2015 Reg. No.:

END OF SEMESTER EXAMINATIONS, APRIL / MAY – 2017

ADVANCED FUNCTIONAL ANALYSIS

SUBJECT CODE : 14P3MA18

MAJOR : M.Sc (Maths) SEMESTER : IV

TIME : 3 HOURS MAX. MARKS: 70

SECTION – A (5 X 4 = 20)

Answer the Following:

1. Prove that in every Hilbert space $H \neq \{0\}$, there exists a total Orthonormal set.

[OR]

2. State and prove the theorem on bounded linear functional.
3. Define fixed point, contraction and write the error estimates.

[OR]

4. Let $T : X \to X$ be a continuous mapping on a complete metric space $X = (X, d)$ and suppose that T^m is a contraction on X for some positive integer m. Prove that T has a unique fixed point.
5. Prove that the space $C[a,b]$ is not strictly convex.

[OR]

6. Let Y be a subspace of the real space $C[a,b]$ satisfying the Haar condition. Given $x \in C[a,b]$, if $y \in Y$ is such that for $x - y$ there exists an alternating set of $n+1$ points, where $n = \dim Y$, prove that y is the best uniform approximation to x out of Y.
7. Define Eigen values, Eigen vectors, Eigen spaces, spectrum and resolvent set of a matrix.

[OR]

8. Give an example of an operator with a spectral value which is not an Eigen value?
9. If X and Y are normed linear spaces, prove that the range $\Re(T)$ of a compact linear operator $T : X \to Y$ is separable.

[OR]

10. If $T : X \to X$ is a compact linear operator and $S : X \to X$ is a bounded linear operator on a normed space X, prove that TS and ST are compact.

SECTION – B (5 X 10 = 50)

Answer the Following:

11. State and prove Riesz's theorem.

[OR]

12. State and prove Baire's category theorem.
13. State and prove Banach fixed point theorem.

[OR]

14. State and prove Picard's Existence and uniqueness theorem.
15. State and prove the Existence theorem for best approximations.

[OR]

16. State and prove Haar uniqueness theorem for best approximation.
17. Prove that all matrices representing a given linear operator $T : X \to X$ on a finite dimensional normed space X relative to various bases for X have the same Eigen values.

[OR]

18. Prove that the resolvent set $e(T)$ of a bounded linear operator T on a complex Banach space X is open and hence the spectrum $\sigma(T)$ is closed.
19. Let (T_n) be a sequence of compact linear operators from a normed space X into a Banach space Y. If (T_n) is uniformly operator convergent, say, $\| T_n - T \| \to 0$, prove that the limit operator T is Compact.

[OR]

20. Let X and Y be normed spaces and X' and Y' the dual spaces of X and Y. Let $T : X \to Y$ be a linear operator. If T is compact, prove that its adjoint operator $T^X : Y' \to X'$ is also compact.

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